Coupled magneto-thermal analysis of split gradient coils in MRI scanners

Fabio Freschi[1,2], *Senior Member, IEEE* Hector Sanchez Lopez[2], Fangfang Tang[2], Maurizio Repetto[1,2], and Stuart Crozier[2]

[1]Department of Energy - Politecnico di Torino, Italy
[2]The School of Information Technology and Electrical Engineering - The University of Queensland, Australia

This paper describes the coupled electromagnetic-thermal analysis of gradient coils in magnetic resonance imaging. This application deserves special attention because the eddy-current analysis of gradient coils is usually performed using filamentary and shell elements, while thermal analysis requires volume elements. The paper aims to present a seamless method to couple the mixed-element discretizations (1d, 2d and 3d) and to project the outputs of eddy currents simulation into the corresponding thermal sources. Special attention is devoted to managing of non-simply connected domains within the integral shell elements formulation.

***Index Terms*—Coupled problems, Eddy currents, Magnetic resonance imaging, Thermal analysis.**

I. Introduction

In magnetic resonance imaging, gradient coils represent one of the essential components since they are used to spatially encode the nuclear magnetic resonance signal. They are designed to produce a linear variation of the axial component of the magnetic field along the Cartesian coordinate in the imaging region. Different values of the magnetic field determines different resonance frequency, according to the Larmor equation [1] $\omega = \gamma B_0$, where B_0 is the magnetic flux density and γ the gyromagnetic ratio, with $\gamma/2\pi = 42.58$ MHz/T. Gradient coils are usually etched on copper sheets with track width ranging from a few to several tens of millimeters. The electromagnetic analysis of gradient coils and their coupling with other conductive structures of MRI scanners is usually performed with integral formulation with shell elements [2], [3], to correctly account of the open boundary nature of the problem. However, when the electromagnetic and thermal analysis are coupled, some modeling and numerical problems arise. Firstly, the integral formulation with shell elements is not suited for closed conductors, like cooling pipes. Secondly thermal simulations require the use of volume meshes, while the thermal sources are modeled as filamentary conductors or shell elements.

The aim of this paper is to present a possible solution for these two problems, making possible a seamless transition between the magnetic and thermal domains.

II. Integral Method

Let us consider a homogeneous unbounded 3d domain with filamentary coils with known currents and a conductive region V. Assuming magnetic quasi-static approximation, the current density $\vec{J}$ in V, is div-free, thus $\vec{J}(\vec{r}) = \nabla \times \vec{T}(\vec{r})$. The study is restricted to the case when V can be approximated by 2d curved surfaces S, i.e. the current density lies on these surfaces. In this case the vector potential can be represented as the scalar stream function ψ: $\vec{J}(\vec{r}) = \nabla \times (\vec{n}(\vec{r})\psi(\vec{r}))$. The stream function is then expanded by nodal shape functions. Taking the curl of these function, after some algebra:

$$\vec{J}(\vec{r}) = \sum_{k=1}^{N} \psi_k \vec{f}_k(\vec{r}) \tag{1}$$

with

$$\vec{f}_k = \nabla \times (\lambda_k(\vec{r})\vec{n}) = \frac{1}{2S}\vec{e}_k \tag{2}$$

$\vec{e}_k$ is the vector corresponding to the edge opposite to node k, and S is the triangle area. Equation (1) is substituted in the total electric field equation: $\vec{E}(\vec{r}) + \mathrm{j}\omega\vec{A}(\vec{r}) + \nabla\varphi(\vec{r}) = 0$ and then a Galerkin scheme is used. The final system has the form:

$$(\mathbf{R} + \mathrm{j}\omega\mathbf{L})\,\boldsymbol{\psi} = -\mathrm{j}\omega\mathbf{a}_\mathrm{s} \tag{3}$$

Where $\mathbf{R}$ is a sparse resistance matrix, $\mathbf{L}$ the dense inductance matrix and $\mathbf{a}_\mathrm{s}$ is the contribution of coils with impressed currents.

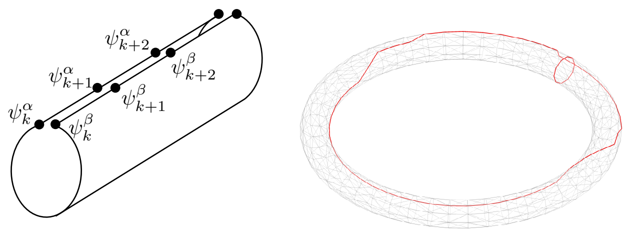
A. Treatment of closed surfaces

Particular attention is required when the surface is closed, for example in case of a torus. Closed surfaces have no boundaries, thus the total current through any cut is zero. To allow a non-vanishing net current through the closed surface, a suitable number of cuts is required, allowing the discontinuity of the stream function. The nodes along the cut are duplicated as shown in Fig. 1a. An additional constraint is imposed, in order to guarantee the current flowing through one side of the cut is equal to that on other side. Using the superscript α for nodes at one side of the cut and β for the other side, these constraints have the form

$$\psi^\alpha_{k+1} - \psi^\alpha_k = \psi^\beta_{k+1} - \psi^\beta_k \tag{4}$$

Constraints like (4) collected in the sparse matrix $\mathbf{B}$ and imposed in the system (3) with the help of Lagrange multipliers λ:

$$\begin{bmatrix} \mathbf{R} + \mathrm{j}\omega\mathbf{L} & \mathbf{B}^\mathrm{T} \\ \mathbf{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\psi} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} -\mathrm{j}\omega\mathbf{a}_\mathrm{s} \\ \mathbf{0} \end{bmatrix} \tag{5}$$



(a) (b)

Fig. 1: (a) The surface is cut along its length, duplicating the unknowns along the cut. (b) Automatically generated cut.

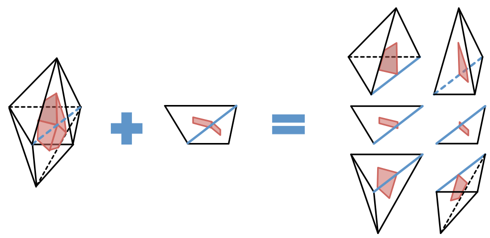


Fig. 2: Assembling of mixed-dimensional element matrix.

1) Automatic generation of cuts

The automatic generation of cuts is a non trivial task. In [4] an algorithm for finding the additional degrees of freedom in a volume integral formulation of the eddy current problem in terms of the electric vector potential is presented. The algorithm used here is its dual, in the sense that it operates directly on the surface mesh and not on the barycentric.

1) create the barycentric dual mesh $\mathcal{B}$ of the original surface mesh $\mathcal{S}$;
2) form the tree $T_{\mathcal{B}}$;
3) select the cotree $C_{\mathcal{B}}$ and recursively remove all the leaves. The loops left represents the cut of the surface mesh $\mathcal{S}$.

Fig. 1b shows the cut generated on the surface of a torus.

III. Thermal coupling

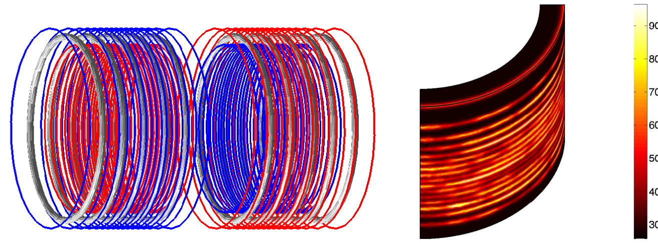
The heat conduction equation is discretized using the finite integration technique, as proposed in [5], [6]:

$$\mathbf{M}_{\rho c}\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{T}+\mathbf{G}^{\mathrm{T}}\mathbf{M}_{\lambda}\mathbf{G}\mathbf{T}=\mathbf{p} \tag{6}$$

where $\mathbf{G}$ the edge-to-node connectivity matrix, $\mathbf{T}$ is the vector of nodal temperatures, and $\mathbf{p}$ is the source term. The thermal capacity matrix $\mathbf{M}_{\rho c}$ and the thermal conductance matrix $\mathbf{M}_{\lambda}$ encode the material properties and the metric of the problem. For this reason the coupling with mixed-dimensional elements, like filamentary coils and shell elements becomes trivial. Making reference to the four tetrahedra sharing the same edge in Fig. 2. The entries of the matrix $\mathbf{M}_{\lambda}$ have the form:

$$m_{\lambda,jk}=\sum_{r}\int_{\tilde{S}^{r}_{j}}\lambda\vec{w}_{k}\cdot\mathrm{d}\vec{S} \tag{7}$$

The summation over the portion of the surface $\tilde{S}_j$ is the circuit equivalent of the *parallel* of thermal conductances. If triangular shell elements are located in between the tetrahedra, their contribution can be calculated independently, and then added, i.e. connected in parallel, to the corresponding elements (7). A similar procedure can be applied for thermal capacity matrix $\mathbf{M}_{\rho c}$.



(a) (b)

Fig. 3: (a) z-coil geometry with pipes. (b) Temperature after 120 s

TABLE I: Comparison of power loss (in watt) between 2d axisymmetric formulation and shell elements with automatic cut generation. Pipes are numbered from bottom to top.

	pipe 1	**pipe 2**	**pipe 3**	**pipe 4**	**pipe 5**
2d axisymm.	7.04	18.22	13.18	4.22	2.64
shell elem.	6.80	17.60	12.65	3.95	2.22

IV. Example

The proposed techniques are tested in the analysis of a split z-coil with cooling pipes. Eddy currents are calculated in the cooling pipes due to a source current of 100 A at 100 Hz. The output is provided as source of the thermal analysis. Table I shows the comparison between a 2d axisymmetric code and the proposed shell elements with automatic cut generation. Fig. 3b shows the temperature map after a heating phase of 120 s.

V. Conclusions

The proposed method allows the automatic coupling between eddy currents and thermal solvers with mixed-dimensional elements. In the full paper the method will be detailed as well as a more accurate analysis on the accuracy will be provided.

References

[1] J. Jin, *Electromagnetic analysis and design in magnetic resonance imaging*. CRC press, 1999.

[2] A. Kameari, "Transient eddy current analysis on thin conductors with arbitrary connections and shapes," *Journal of Computational Pysics*, vol. 42, pp. 124–140, 1981.

[3] H. Sanchez Lopez, M. Poole, A. Trakic, F. Freschi, E. Smith, J. Herbert, M. Fuentes, Y. Li, F. Tang, M. Repetto, and S. Crozier, "Modal analysis of currents induced by magnetic resonance imaging gradient coils," *IEEE Transactions on Magnetics*, vol. 50, no. 2, 2014.

[4] G. Rubinacci and A. Tamburrino, "Automatic treatment of multiply connected regions in integral formulations," *IEEE Transactions on Magnetics*, vol. 46, no. 8, pp. 2791–2794, 2010.

[5] M. Clemens, E. Gjonaj, P. Pinder, and T. Weiland, "Numerical simulation of coupled transient thermal and electromagnetic fields with the finite integration method," *IEEE Transactions on Magnetics*, vol. 36, no. 4, pp. 1448–1452, 2000.

[6] P. Alotto, F. Freschi, and M. Repetto, "Multiphysics problems via the cell method: The role of tonti diagrams," *IEEE Transactions on Magnetics*, vol. 46, no. 8, pp. 2959–2962, Aug 2010.